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EXPLOSION METHOD OF PREVENTING COLLISIONS OF ASTEROID-COMET BODIES WITH THE EARTH IN THE CASE OF THEIR LATE DETECTION

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Based on the model concepts of destruction of the substance of a celestial body in the shock wave initiated by a strong explosion of weak penetration, the radius of the destroyed region, the ejected mass, and the recoil momentum have been evaluated. The values of the charges necessary for destroying completely bodies of different size and composition or withdrawing bodies from the Earth for a required distance have been determined. The efficiencies of the explosion and sublimation methods of changing orbits for the case where the hazardous bodies are comets have been compared. Problems on improving the efficiency of the explosion method of action on a hazardous body due to the high relative velocity of impact of this body with the charge-carrying rocket have been discussed.

Introduction. Small planets (asteroids) and comets approaching the Earth represent a real hazard as possible sources of regional and, at worst, global disasters [1]. A strong explosion is the most efficient method of diminishing the hazard [2] in the case of late detection (in the near "reaches" of the Earth) of hazardous space objects (HSOs). The required charge power may prove to be extremely high for complete destruction of a large body; therefore, it seems more advantageous to transfer it to a "safe" trajectory using explosion action. The physical processes with allowance, for example, for the radiant transfer in the evaporated substance of an HSO and for its real equation of state can, apparently, be modeled most accurately using only numerical methods [3–5]. Below, we determine the conditions of explosion action on an HSO that are necessary for withdrawing the body or destroying it: the value of the charge, the range of action, etc. Analogously to [6, 7], where the computational formulas for determining the values of the energy release, the radius of the region destroyed, and the withdrawal of a hazardous body from the Earth have been obtained, below we use model concepts developed in [8–11].

Formulation of the Problem. In most formulations of problems on explosion action on an HSO, one disregards the influence of the relative velocity of a charge and this body on the parameters of a shock wave propagating in it and causing destruction. The efficiency of the action is improved if the charge is delivered to the HSO by special rockets with a high relative velocity. Therefore, it seems of interest to evaluate the influence of this velocity on the parameters of the shock wave and hence on the efficiency of energy release in explosion. These evaluations are given below, and preliminary data on this problem are contained in [12]. It is noteworthy that not only are the problems mentioned important for the problem of asteroid-comet hazard but they are also topical from a purely cosmogonical viewpoint, since explosions and collisions between celestial bodies occur in space at all times.

We will assume that before the explosion at the instant $t = 0$ the HSO has the shape of a sphere and moves toward the Earth with velocity V_0 along a straight line (Fig. 1) connecting the centers of mass (points O and A) the distance between which is r_0. Clearly, such a straight trajectory is the least favorable from the viewpoint of the value of the charge necessary for deflecting the HSO.

We will assume that the vector of the explosion force $\mathbf{F}$ applied to the HSO at an angle ψ to $\mathbf{r}_0$ passes through the center of the body, so that the torque is absent, and the disturbed trajectory lies in the plane formed by the vectors $\mathbf{r}_0$ and $\mathbf{F}$. For the sake of convenience, we thereafter use two coordinate systems: a Cartesian system (x, y) with the X axis passing, at the instant of explosion, through the HSO center, i.e., coincident with the vector $\mathbf{r}_0$ in direction, at the step of explosion (first step), and a polar system (r, φ), in which the angle φ is counted off from the

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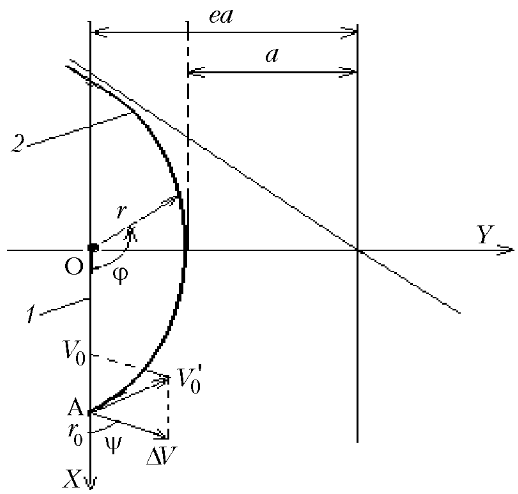


Fig. 1. Diagrams of initial (1) and disturbed (2) motion of an HSO (explosion action on the HSO at point A); O, origin of coordinates, coincident with the center of the Earth; A, point of application of explosion momentum (with the coordinate r_0 on the X axis)).

indicated direction $\mathbf{r}_0$, at the step of passive motion (second step). We preliminarily consider the case of collision (in the absence of explosion).

Problem on Collision. In general form, the problem is described by the equations

$$\ddot{r} - r\dot{\varphi}^2 = -\mu/r^2 , \tag{1}$$

$$r^2\dot{\varphi} = c , \tag{2}$$

where $\mu = \gamma M_E$; for the straight motion we have

$$\dot{\varphi} = c = 0 , \tag{3}$$

and the initial conditions have the form

$$r\Big|_{t=0} = r_0 , \quad \dot{r}\Big|_{t=0} = V_0 < 0 . \tag{4}$$

Under actual conditions, when an HSO approaches the Earth from "infinity," the energy constant is $h = \frac{V_0^2}{2} - \frac{\mu}{r_0} > 0 \left(\alpha = \frac{V_0^2 r_0}{2\mu} > 1\right)$; therefore, the solution of the problem corresponds to motion along a degenerate hyperbola for which, integrating (1) and (2), we can obtain, with account for (3) and (4), a relationship between the dimensionless coordinate and time:

$$\bar{t} = \frac{\sqrt{\alpha}}{2(\alpha-1)\sqrt{\alpha-1}} \left\{ \frac{2}{\delta} [\sqrt{\delta+1} - \sqrt{\bar{r}(\delta+\bar{r})}] + \ln \frac{(\sqrt{\delta+1}-1)(\sqrt{\delta+\bar{r}}+\sqrt{\bar{r}})}{(\sqrt{\delta+1}+1)(\sqrt{\delta+\bar{r}}-\sqrt{\bar{r}})} \right\} , \tag{5}$$

where $\bar{t} = t/t_0$, $\bar{r} = r/r_0$, $t_0 = r_0/V_0$, and $\delta = a/r_0 = (\alpha-1)^{-1}$; $\alpha = \mu/h$ is the value of the large semiaxis of the degenerate hyperbola. Disregarding the radius of the Earth compared to the initial distance to the HSO, we obtain from (5) the total dimensionless time of flight of the body before the collision:

$$\bar{t}_{r_0} = \bar{t}\,\big|_{\bar{r}=0} = \frac{\sqrt{\alpha}}{2(\alpha-1)\sqrt{\alpha-1}} \left(\ln \frac{\sqrt{\delta+1}-1}{\sqrt{\delta+1}+1} + \frac{2}{\delta}\sqrt{\delta+1} \right). \tag{6}$$

For the quantities r_0 and V_0 of interest, we usually have $\alpha >> 1$ ($\delta << 1$); therefore, computing the limit of expression (6), we obtain natural estimates characteristic of the HSO motion with a constant velocity (when $\alpha >> 1$): $\bar{t}_{r_0} = 1$ (or $t_{r_0} = r_0/V_0$). For example, we have $t_{r_0} = 0.92$ day for $r_0 = 1$ mln km and $V_0 = 12$ km/sec (when the heliocentric velocities of the Earth and the HSO are coincident in direction), $t_{r_0} = 0.26$ day for the same r_0 and $V_0 = 42$ km/sec (the velocity vectors are perpendicular), and $t_{r_0} = 0.15$ day for $V_0 = 72$ km/sec (the velocities are opposite in direction).

Explosion Method of Diminishing an Asteroid-Comet Hazard. In the first step, in the process of explosion, the body becomes depleted of its initial mass, which must be taken into account in determining its velocity V_0' immediately after the explosion. The HSO motion is described by variable-mass equations (Meshcherskii-type equations), which, in Cartesian coordinates, have the form

$$M\frac{dV_X}{dt} = \frac{dM}{dt}u_X - Mg\frac{x}{r}, \quad M\frac{dV_Y}{dt} = \frac{dM}{dt}u_Y - Mg\frac{y}{r},$$

$$g = g_E\,(R_E/r)^2, \quad r = \sqrt{x^2+y^2}\,. \tag{7}$$

By virtue of the short duration of this step, the HSO coordinates here change only slightly compared to the initial distance r_0 to the Earth's center. Therefore, it can be approximated that we have $x \approx r_0$, $y \approx 0$, and $g \approx g_{r_0} = g_E(R_E/r_0)^2 = \text{const}$ over the period of action of the explosion (the vector $\mathbf{r}_0$ is assumed to be directed along the X axis). Dividing (7) into M, integrating with respect to t, and discarding the small term $g_{r_0}t_f$ in the first equation, for the projections of the HSO velocity at the end of the first step we obtain

$$V'_{X_0} = V_{X_0} + \Delta V_X = V_{X_0} + \Delta V\cos\psi = V_{X_0} + u_X\ln\frac{M_f}{M_0} = V_{X_0} + u\cos\psi\ln\frac{M_f}{M_0}, \tag{8}$$

$$V'_{Y_0} = V_{Y_0} + \Delta V_Y = V_{Y_0} + \Delta V\sin\psi = V_{Y_0} + u_Y\ln\frac{M_f}{M_0} = V_{Y_0} + u\sin\psi\ln\frac{M_f}{M_0}, \tag{9}$$

where V_{X_0} and V_{Y_0} are the projections of the HSO velocity at the initial instant of explosion and u_X and u_Y are the projections of the average velocity u of ejection of the mass M_1 related to the initial M_0 and final M_f masses as

$$M_f = M_0 - M_1\,. \tag{10}$$

The most accurate investigations of explosion action with allowance for the physical processes in the stages of energy release and thermal and shock waves, up to the elastoplastic stage, are only possible by numerical methods. The results of calculating the action of a nuclear explosion on HSOs and soils under ground conditions have been given, for example, in [3–5]. Below, we roughly evaluate the ejected mass M_1 and the average velocity u without allowance for the ejection occurring in the earlier (thermal) stage. Based on the model concepts [6, 13] of the destruction of a body in a spherical shock wave propagating in the HSO material for the case of a high-velocity impact of a missile with a charge upon the body and the explosion of the charge, from energy considerations we can obtain

$$E_0\chi = 8\pi\frac{n+1}{n-1}\eta\rho_0\varepsilon_d R_f^3\,. \tag{11}$$

We have taken $\rho_0 = 7850$, 3400, and 500 kg/m^3 for iron and stony HSOs and for the substance of comet nuclei respectively. At pressures higher than 10^6 kg/cm^2, we have $n = 3$ and $\eta = 1/6$ [9, 13]. According to [13], ε_d is the energy of destruction "... when a medium will break down into small elastic solid particles which are similar to a quasigas in properties ..." (for iron and stony bodies we can take $\varepsilon_d = 10^5$ J/kg [13]; for comet nuclei, judging from the energy of phase transition of ice, this quantity is at least three times lower, i.e., $\varepsilon_d = 0.3\cdot10^5$ J/kg). In connection with the existing projects of penetration of a charge into the HSO substance (see, e.g., [14]) and for the reasons indicated in [6], in subsequent calculations, we take $\chi = 0.30$ for iron HSO and $\chi = 0.35$ for stony ones; $\chi = 0.4$ is taken

TABLE 1. Energy of Complete Destruction of HSOs (E_d, Mtons) of Different Size and Composition

d_0, m	Stone	Iron	Cometary Ice
100	1.945	5.238	0.075
200	15.557	41.906	0.600
300	52.506	141.432	2.027
400	124.459	335.246	4.804
500	243.084	654.777	9.384
600	420.048	1131.454	16.215
700	667.021	1707.658	25.749
800	995.670	2681.965	38.436
900	1417.664	3818.658	54.726
1000	1944.669	5238.214	75.070
1500	6563.257	17,678.971	253.361
2000	15,557.354	41,905.709	600.560

for cometary ice. For these values of the physical parameters, Table 1 gives the energies E_d of complete destruction of bodies of different size and model composition; these energies have been calculated from formula (11) when $R_f = d$. It is seen that the destruction of large-size bodies (particularly stony and iron ones) will require multiple explosions, i.e., such a method of neutralizing the hazard is not optimum.

Assuming that the entire mass M_1 evaporated and broken down by the shock wave will be ejected and integrating with respect to the part of the surface S of the shock wave covering a spherical HSO of diameter d, we obtain

$$M_1 = \rho_0 \int_0^{R_f} S dr = 2\pi\rho_0 \int_0^{R_f} r^2 \left(1 - \frac{r}{d}\right) dr = \frac{\pi}{2} \rho_0 \beta R_f^3 , \tag{12}$$

where $\beta = \frac{4}{3} - R_f d$ is the destruction parameter.

A certain part ζ of the total energy E_0 of explosion is transferred, as kinetic energy, to the substance M_1 ejected with a rate u and to the body of mass M_f remaining after the explosion and obtaining the increment in velocity ΔV:

$$2\zeta E_0 = M_1 u^2 + M_f \Delta V^2 . \tag{13}$$

From (8)–(13), it follows that

$$\Delta V = -\frac{u^* \ln m}{\sqrt{\beta + \frac{\beta m}{1-m} (\ln m)^2}} , \tag{14}$$

where $u^* = \sqrt{\frac{32\zeta(n+1)\,\eta\varepsilon_d}{\chi(n-1)}}$ and $m = \frac{M_f}{M_0} = 1 - 3\beta\left(\frac{4}{3} - \beta\right)^3$; $0 < m < 1$. The relative ejected mass is subsequently computed from the formula $M_1/M_0 = 1 - m$. It is assumed that ζ/χ = const; in particular, we have $\zeta = 0.25$ for stony HSOs.

On the other hand, we can determine the vector $\Delta\mathbf{V}$, considering the second step of disturbed motion calculated from the safety conditions of a passive HSO flight by the Earth in the Kepler orbit after the explosion. The motion of the body in this step is also described by Eqs. (1) and (2), but now we have $c = r_0 \Delta V \sin\psi$, i.e., $c \neq 0$ now. Thus, initial conditions have the form

$$r\big|_{t=0} = r_0\,,\quad \dot{r}\big|_{t=0} = V'_{X_0}\,,\quad r_0\dot{\varphi}\big|_{t=0} = V'_{Y_0} = \Delta V_Y\,. \tag{15}$$

The integral of system (1) and (2) is expressed in this case by the well-known formula

$$r = \frac{p}{1 + e\cos(\varphi - \varphi_0)}\,. \tag{16}$$

Disregarding the HSO mass compared to the Earth's mass and the radial component of the vector $\mathbf{\Delta V}$ compared to the initial velocity $\mathbf{V}_0$ of the body, with account for (15) we obtain

$$p = \frac{c^2}{\mu} = \frac{(r_0\Delta V\sin\psi)^2}{\mu}\,,\quad e = \sqrt{1 + \frac{2p^2}{c^2}\left(\frac{V_0'^2}{2} - \frac{\mu}{r_0}\right)} \approx \sqrt{1 + \left(\frac{r_0V_0\Delta V\sin\psi}{\mu}\right)^2}\,. \tag{17}$$

The minimum distance to the HSO from the center of the Earth is equal to

$$r_{\min} = \frac{p}{1+e} = \frac{a\lambda^2}{1 + \sqrt{1+\lambda^2}}\,, \tag{18}$$

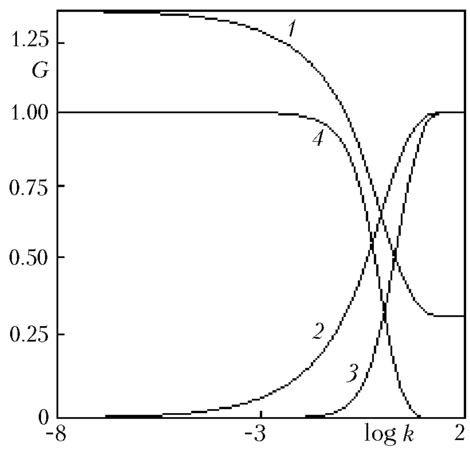
where $a = \mu/V_0^2$ and $\lambda = r_0V_0\Delta V\sin\psi/\mu$. Solving the algebraic equation (18) for λ and using (15), we find the basic relation determining the necessary conditions of explosion:

$$k = -\frac{\sin\psi\ln m}{\sqrt{\beta + \dfrac{\beta m}{1-m}(\ln m)^2}}\,. \tag{19}$$

where $k = \dfrac{\Delta V}{u^*} = \dfrac{\sqrt{\mu r_{\min}}}{r_0u^*}\sqrt{2 + \dfrac{r_{\min}V_0^2}{\mu}}$ is the kinematic parameter dependent on the initial conditions, the characteristics of motion of an HSO, and its physical properties. It can be shown that the quantity k represents the dimensionless (per unit mass) mechanical momentum $k = I_f/(mI_0)$ which is imparted to the body remaining intact (I_f is the true value of the total momentum of this body and $I_0 = M_0u^*$ is the characteristic quantity). In the particular case of an infinitesimal influence of the Earth, series expansion of the expression for k yields $k = r_{\min}V_0/(r_0u^*) = r_{\min}/(t_0u^*)$, which coincides with the results of [15, 16] for a constant HSO mass. The first and second terms under the radical sign in the denominator of (19) correspond respectively to the kinetic energy of the substance ejected in explosion and of the body undestroyed. As is easily shown, their maximum ratio (for $m = 0.089$) is no more than 0.57, so that the denominator in (19) does not exceed $1.25\sqrt{\beta}$; therefore, for rough evaluations we can omit the factor and can set

$$k \approx -\frac{\sin\psi\ln m}{\sqrt{\beta}}\,.$$

Expression (19) written in dimensionless form determines the geometric and energy similarity laws characterizing the degree of explosive destruction of bodies of different diameter as a function of the known initial conditions and the required deflection of the orbit ("miss"), which are related by the parameter k. Figure 2 plots the destruction parameter β, the relative size of the destroyed region R_f/d, the ratio $E_0 = E_d = (4/3 - \beta)^3$ of partial and complete destruction, and the fraction m of the undestroyed mass versus k. The asymptotic values $\beta = 4/3$ and $\beta = 1/3$ correspond to the absence of the destruction of the HSO ($m = 1$) and its complete destruction ($m = 0$). An analysis shows that the transfer of energy and momentum to an undestroyed body is optimum in character under certain conditions. This is clear, for example, from Fig. 3, which plots the following relative quantities as functions of



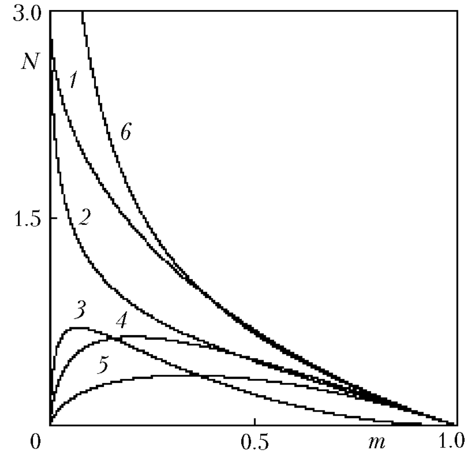


Fig. 2. Characteristics of the explosion G vs. values of the kinematic parameter k: 1) $G = \beta$; 2) R_f/d; 3) E_0/E_d; 4) m.

Fig. 3. Characteristics of motion of the substance ejected in explosion and the remaining body N vs. degree of destruction m: 1) $N = E_0'$; 2) E_1; 3) E_f; 4) E_f/E_1; 5) I_f'; 6) k.

TABLE 2. Results of Calculation of the Charge Power (E_0, Mtons) Necessary for Withdrawing Stony HSOs and the Residual Mass of the Body (m) under Different Initial Conditions

$r_0 \cdot 10^{-3}$, km	L, km	$V_0 = 15$ km/sec						$V_0 = 30$ km/sec					
		d, m					m	d, m					m
		100	200	300	500	1000		100	200	300	500	1000	
10	200	1.9	15.2	51.5	238.3	1906.7	0.0003	1.9	15.5	52.4	242.9	1942.8	10^{-6}
25	200	1.6	13.0	43.9	203.6	1628.6	0.0183	1.6	14.6	49.3	228.3	1826.6	0.0025
50	200	1.3	10.0	34.0	157.6	1261.3	0.0899	1.6	12.4	42.0	194.6	1557.4	0.0277
100	500	0.9	6.8	23.1	107.0	856.4	0.2436	1.2	9.5	32.2	149.2	1194.3	0.1096
300	750	0.3	2.7	9.1	42.4	339.4	0.5945	0.6	4.5	15.3	70.9	567.3	0.4136
500	1000	0.2	1.7	5.6	26.0	208.4	0.7241	0.4	2.9	9.8	45.5	364.0	0.5725
1000	1750	0.1	0.9	3.0	13.7	110.4	0.8387	0.2	1.6	5.4	24.7	198.3	0.7350
1500	2500	0.1	0.6	2.1	9.7	77.8	0.8809	0.1	1.1	3.8	17.7	141.9	0.7996
5000	5000	0.03	0.2	0.8	3.5	27.7	0.9532	0.1	0.4	1.4	6.4	51.5	0.9176

m: $E_0' = E_1 + E_f$, the total explosion energy necessary for destroying the HSO completely (left-hand side of Eq. (13)), E_1 and E_f, the kinetic energies transferred to the ejected substance and the remaining body (1st and 2nd terms on the right-hand side of (13)), E_f/E_1, their ratio, and $I_f' = I_f/I_0$ and k, the total and specific momenta imparted to the remaining body. The true values of the energy E_0, E_1, and E_f in Fig. 3 are referred to the characteristic quantity $M_0 u^{*2}/2\zeta$. Curves 3, 4, and 5 have maxima of 0.717, 0.648, and 0.366 respectively for m values of 0.069, 0.219, and 0.332.

The order of calculations is as follows (there can be other computational schemes). First we determine, from formula (11), the explosion energy E_d necessary for destroying completely a body of prescribed diameter d. This diameter and the E_d value obtained are those reference values for further computations. Next, assuming the kinematic parameter k to be known (since r_0, r_{min}, and V_0 are known), we find the most important relative quantities partly shown in Fig. 2 by numerical solution of the algebraic equation (19). Their multiplication by the corresponding reference values which are characteristic ones yields the physical parameters sought.

Table 2 gives results of evaluations of the charge power E_0 ensuring the deflection of stony HSOs for a prescribed distance $L = r_{min} - R_E$ from the Earth's surface and the fraction m of the residual mass (only the data for V_0

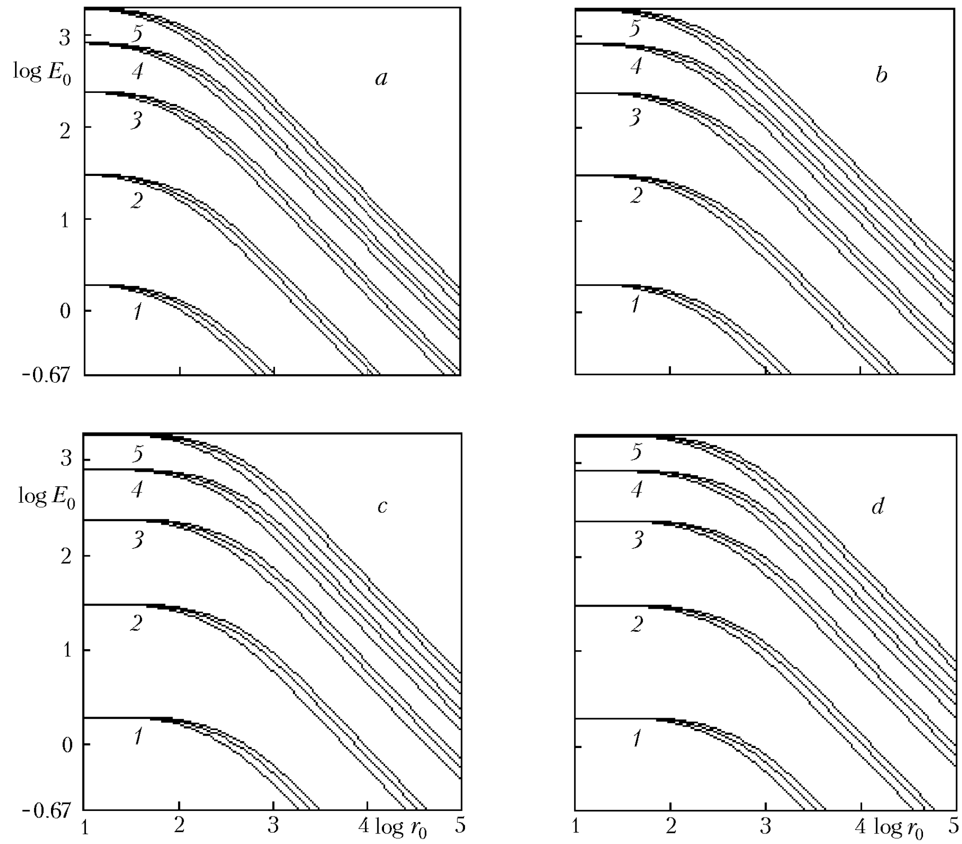


Fig. 4. Explosion energies E_0 necessary for withdrawing an HSO vs. initial distance r_0 to the center of the Earth for different velocities [a) V_0 = 15; b) 30; c) 50; d) 70 km/sec] and size of the body [1) d = 100; 2) 250; 3) 500; 4) 750; 5) 1000 m; the lower curves in the bundle correspond to $L = 0.5R_E$, the middle curves correspond to $L = R_E$, and the upper ones correspond to $L = 1.5R_E$]. E_0, Mtons; r_0, thousand km.

= 15 and 30 km/sec are tabulated). The optimum direction of explosion momentum (when L is maximum) has been selected, which corresponds to the angle $\psi = \pi/2$.

It is clear from the table that if the HSO with an initial velocity of 30 km/sec and a diameter of 300 m is exposed to an explosion of 5.4 Mtons at a distance of 10^6 km (approximately three distances to the Moon), the deflection of the trajectory will be no less than 1700 km. The residual relative mass will be equal to ~0.74. Larger bodies are more safely "deflected" for much longer distances. Thus, for an HSO of diameter 1 km (for the same value of V_0) to fly by the Earth at a distance of 5000 km, the required charge power must be 51 Mtons, and the explosion must be initiated at a distance of $5 \cdot 10^6$ km. The residual mass m is equal to 0.92, i.e., the relative mass loss in ejection proves to be smaller than that in the previous case. The dependences $E_0(r_0)$ in a wider range of variation are plotted on a logarithmic scale in Fig. 4 for different initial data (for V_0 = 15, 30, 50, and 70 km/sec). The horizontal asymptotic portions of the curves in their left-hand part (when r_0 are comparatively small) correspond to the energy of complete destruction of the HSO. These values change with size of the body d but are independent of the withdrawal r_{min} and the velocity of the body V_0. Otherwise, for large r_0 (small k), the relative values of the mass loss and the partial-destruction energy are minimum (see Fig. 2) and we can show, using the expansion of the right-hand side of (19), that $E_0 \to kE_d/2\sqrt{3} \sim 1/r_0$, i.e., the curve on the right in the figure tends to a straight line having a negative slope.

It should be noted that although most of the estimates are given here for stony bodies, the calculations can easily be performed for other materials, too. Recalculation mainly holds true just for the characteristic quantity E_d, which is most strongly dependent on the physical properties of an HSO.

Comparative Analysis of Explosion and Sublimation Methods of Withdrawing Comets. In the case of early detection where the hazardous objects are comets at a large distance from the Earth (moving in the solar field — early detection) the sublimation method of changing orbits is more promising; this method is based on artificial enhancement of cometary activity, which results in the additional jet thrust [17]. We compare the sublimation and explosion methods of withdrawing in the opposite case — that of late detection of hazardous comets (motion in the solar field). Let us consider each method successively, making two simplifying assumptions: we will disregard terrestrial attraction (this case has been considered above in the absence of action) and mass loss in the process of acting.

Explosion Method. As has been noted earlier, the first assumption (absence of gravity) means that the kinematic parameter in Eq. (19) is

$$k = \frac{r_{\min} V_0}{r_0 u^*} = \frac{r_{\min}}{t_0 u^*} = \frac{\Delta V}{u^*}, \tag{20}$$

i.e., the comet nucleus moves by the Earth along the straight line.

Based on the second assumption (constancy of mass) we can set $m = 1 - \Delta m$, where $m \ll 1$ (this inequality is not universally true). Therefore, expanding the right-hand side of (19) in series in Δm accurate to the first term and taking into account that $\beta \to 4/3$ in the limit, we obtain

$$k = 2\sqrt{3} \sin \psi \, (R_f/d)^3 = 2\sqrt{3} \sin \psi \, (E_0/E_d) . \tag{21}$$

Here we have used the similarity relation $(R_f/d)^3 = E_0/E_d$.

Taking into account that we have $E_d = E_0|_{R_f=d}$ according to formula (11), we find an approximate expression for the "miss" in the explosion method of withdrawing comets (asteroids) based on (20) and (21):

$$r_{\min} = \frac{\sqrt{3}\, \zeta t_0 E_0}{\pi u^* \rho_0 R_n}, \tag{22}$$

where $R_n = d/2$ is the radius of the spherical comet nucleus.

Sublimation Method. Once the cometary activity has been initiated (according to the first assumptions), the nucleus is acted upon by a single force F due to the sublimation effect. The distance from the nucleus to the Sun can be assumed to be constant over the entire period of expected "fall" of the body on the Earth and to be equal to 1 au. Consequently, the temperature T of the nucleus and the pressure P_v of vapor efficiency at the subsolar point are constant, too. Their values at the distance indicated are equal to T_0 = 205 K and P_{v0} = 0.35 N/m^2, as the calculations show [17, 18]. The dependence of these quantities and the force F on the angle θ between the direction to the Sun and the normal to the nucleus surface is the most pronounced near the angle $\theta = 90^{\circ}$ [18]; at least at $0 \le \theta \le \theta_1 = 85^{\circ}$, the above quantities can be assumed to be constant and equal to their values at the subsolar point. The effective radius of the nucleus is equal to $R_1 = R_n \sin \theta_1 = 0.9962 R_n$, i.e., it differs from its true value R_n only slightly. Under the assumptions made and with allowance for the constant force F and mass M_0 of the nucleus, from the equation of motion we find the sought deflection from the Earth: $r'_{\min} = 0.5 F t_0^2 / M_0$, where $t_0 = r_0/V_0$. With allowance for this fact, we obtain $F \approx P_{v0} S_n$, $M_0 = 4\pi\rho_0 R_n^3/3$, and

$$r'_{\min} = \frac{3 P_0 t_0^2}{8 \rho_0 R_n}. \tag{23}$$

Comparison of the Methods. The efficiency of each method can be judged from the ratio of the ranges of withdrawal (22) and (23) under equal conditions:

$$\frac{r'_{\min}}{r_{\min}} = \frac{\sqrt{3}\, \pi R_n^2 P_0 u^* t_0}{8 \zeta E_0}. \tag{24}$$

From formula (24), it is seen that the sublimation method is more advantageously used for withdrawal of comparatively large nuclei (with a large area $S_n = \pi R_n^2$), which are at a fairly large distance from the Earth or fly slowly to it ($t_0 = r_0/V$ is large). In this case we have $r'_{min}/r_{min} > 1$. The methods compared are equally accurate if $r'_{min}/r_{min} = 1$. The explosion method of withdrawing is more efficient when $r'_{min}/r_{min} < 1$. Formal solution of the equation $r'_{min}/r_{min} = 1$ at different reasonable reserves of time t_0 (and the velocity of the body V_0 = 30 km/sec) shows that the advantages of the explosion method manifest themselves for comparatively low-power charges. For example, the nucleus radius is R_n = 1.3 km even for E_0 = 1 kg and t_0 = 0.1 day (r_0 = 0.26 mln km), which corresponds to the size of actual comets. However, the ranges of withdrawal using such charges are negligible (in the above case formula (20) yields $r_{min} = 8\cdot10^{-3}$ km, which is much smaller than the Earth's radius). Deflection of this body for an acceptable distance requires much more intense explosions: based on (20), the required charge power for deflection, e.g., for a distance of 200 km from the Earth's surface, is 822 Mtons.

Effect of Enhancement of Shock Waves Formed in Celestial Bodies Hazardous to the Earth in the Case of Their Explosive Destruction by a Moving Charge. Let us consider the influence of the relative velocity of a moving charge on shock-wave processes occurring in the case of explosion action on a celestial body hazardous to the Earth. We show that allowance for this fact makes it possible to increase the energy release and consequently to improve the efficiency of action to an extent dependent on the body's size.

The total explosion energy E_0 is related to the pressure P on the front of a shock wave propagating from the epicenter of the explosion by the well-known relation [6, 9, 10, 13]

$$E_0 = \frac{4\pi}{n-1}\eta P R^3 , \tag{25}$$

where R is the distance from the epicenter of the explosion.

Formula (25) characterizes the attenuation of the shock wave in an unbounded medium in its propagation from the epicenter. K. P. Stanyukovich, who studied the phenomenon of attenuation of a shock wave upon the impact of a meteorite on the surface of a planet (Moon), i.e., under conditions similar to those considered, took no account of a certain weakening of the shock wave due to the scatter of the gaseous explosion products to vacuum, assuming that the wave propagates just in the same manner as in the case of a strong explosion in an unbounded medium. The possibility of such an assumption has been substantiated by calculations in [11]. Showing that only a very small part of the substance of the medium flows out of the explosion bowl over the period of traversal of the shock wave, Zel'dovich et al. inferred that "... the outflow of a gas from the shock-wave front to vacuum makes the shock wave weaker only slightly compared to the explosion in an unbounded medium."

The velocity of the shock-wave front D is related to the front pressure by the dependence [9, 11]

$$D^2 = \frac{P}{\rho_0^2}\left(\frac{1}{v_0 - v}\right) = \frac{P}{\rho_0}(n+1) . \tag{26}$$

Expressing the pressure P by D^2 from relation (26) and substituting it into the formula for E_0, we obtain

$$E_0 = \frac{4\pi}{n^2-1}\eta\rho_0 R_0^3 D^2 , \tag{27}$$

where R_0 is the effective radius of the exterior charge surface.

Upon the penetration of a moving charge into a stationary body and its explosion, the velocity of propagation of the shock wave over the body in the direction of motion increases by the value of its velocity, whereas in the case of the opposing HSO motion it increases by the total value w of the velocities of the charge and body. This makes it possible, based on formula (27), relating the explosion energy to the shock-wave velocity, to introduce the coefficient ξ characterizing the increase in the explosion energy in opposing motion of the charge and the body. We assume that the shock wave has a nearly spherical shape and propagates in the direction of external normals to its surface. In this case, the coefficient ξ of increase in the energy can be calculated as

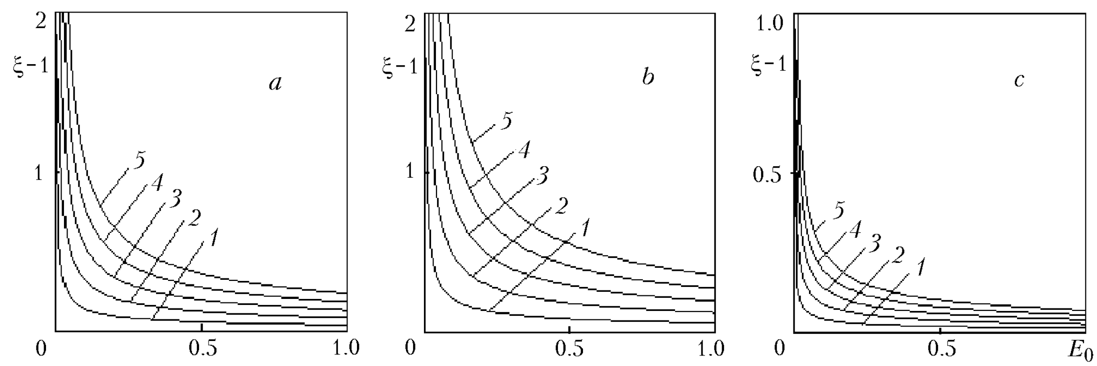


Fig. 5. Increase in the energy release $\xi - 1$ in HSOs of different composition [a) stone; b) iron; c) ice] vs. explosion power E_0 for different relative velocities of the charge: 1) w = 15; 2) 30; 3) 45; 4) 60; 5) 75 km/sec. E_0, ktons.

$$\xi = \frac{E_0'}{E_0} = \frac{1}{2\pi R_0^2} \int_0^{\pi/2} 2\pi R_0^2 \sin\omega \left(\frac{D + w\cos\omega}{D}\right)^2 d\omega = \int_0^{\pi/2} \sin\omega\,(1 + \kappa\cos\omega)^2\, d\omega\,, \tag{28}$$

where $\kappa = w/D$ is the relative velocity, which, with account for (27), is transformed to the form

$$\kappa = w\sqrt{\frac{4\pi\eta\rho_0 R_0^3}{E_0\,(n^2 - 1)}}\,. \tag{29}$$

Computing the integral (28), we obtain the final expression for the coefficient of increase in the explosion energy:

$$\xi = 1 + \kappa + \frac{\kappa^2}{3}\,. \tag{30}$$

From expressions (29) and (30), it is seen that the higher the velocity and radius of the charge and the density of the HSO substance and the lower the energy release, the stronger the enhancement of the shock wave. We note that the explosion energy increases for a high velocity of collision of the charge with the HSO body due to the fact that the energy of the charge is liberated when it bites deeper into the body [6, 13]. Evaluating by formulas (25)–(29), we should remember that only part of the energy χ dependent on explosion conditions, for example, on the degree of penetration of the charge, and not the entire energy E_0 goes into the shock wave in explosion, as a rule.

We evaluate the coefficient of enhancement of the shock wave for different cases. Let us consider a charge with parameters E_0 = 10 ktons and R_0 = 1 m. For stony meteorites, from formulas (29) and (30) we obtain κ = 0.256 and ξ = 1.278 for the relative velocity w = 60 km/sec and κ = 0.128 and ξ = 1.133 for w = 30 km/sec. For the lower explosion energies used for destruction of comparatively small celestial bodies, the enhancement of the shock wave is even larger. According to the results of [6], the charge energy E_0 = 0.014 kton (with utilization factor χ = 0.4) is necessary for destroying, for example, an ice nucleus of density 500 kg/m^3 and diameter 60 m (the Tunguska meteorite, which is the nucleus of a minicomet (in accordance with the most substantiated hypothesis) had nearly the same size). If we take the relative velocity w = 60 km/sec, we will have κ = 0.142 and ξ = 1.149. Figure 5 gives the quantity $\xi - 1$ characterizing the increase in the energy release in three of the most typical media (stone, iron, and cometary ice) as a function of the explosion power E_0 for different values of the relative charge velocity w. Their analysis enables us to infer that in the case of consideration of medium-size bodies (of the Tunguska-meteorite type) and smaller objects representing a real asteroid-comet hazard the effect of enhancement of shock waves in explosion of a moving charge is fairly large (of the order of several units); consequently, it must be taken into account in evaluating the force action of the bodies indicated. As is clear from the figure, iron, whose density is higher, has the largest coefficient of enhancement.

Conclusions. The calculations carried out show that the explosion energy necessary for transferring an HSO to a safe trajectory is much lower than that necessary for destroying the body completely; the charges required for withdrawal of iron HSOs are nearly 2.7 times larger (70 times smaller for ice nuclei) than those required for withdrawal of stony bodies. Deflection of very large HSOs in the case of very small distances from them to the Earth will, possibly, require a number of rockets, not a single one, launched at short time intervals. A comparison of the explosion and sublimation methods of withdrawing comets moving in circumterrestrial space shows that the nuclei of typical size are more efficiently deflected by explosion. However, the values of the "misses" obtained do not exceed the Earth's radius or several values of it even in the case of multiple explosions, which is the drawback of near-range interception. The use of this method at a large distance — for the HSO motion in the solar field, when we can obtain much wider ranges of withdrawal (including those of no less than 1 mln km) and safer explosion conditions seems more promising and reliable. However, the sublimation method of withdrawing is more efficient at such distances from the Earth in the case of comets [17].

The physical model used allows only for the disperse destruction of the HSO substance in the shock wave and for the ejection of the mass destroyed but it does not take into account possible phenomena of large-block fragmentation, which are seemingly of importance for a large mass loss. However, since these phenomena are characteristic of the later stage of propagation of the shock wave (following the evaporation stage), when its intensity becomes noticeably weaker, the process of fragmentation of the body remaining after the evaporation will occur in the presence of additional (evaporative) momentum; therefore, fragments will also receive the component of velocity in the necessary direction, which will diminish the probability that they will fall on the Earth. This problem is to be studied in greater detail in the future. Certain evaluations with allowance for the above factors have been performed earlier (e.g., in [2]). We note that the accuracy of calculations is largely dependent on the physical constants used, whose real values are unknown at present. Their determination, in particular, can be the objective of future space missions to asteroids and comets.

The described method of improving the efficiency of explosion action can successfully be used in any variants of detection of HSOs. The high velocity of an HSO in relation to the velocity of the charge contributes, as a rule, to its realization. In the case of late detection where the reserve of time is limited, the high velocity of the rockets themselves is a natural factor increasing energy release, too (which also contributes to the penetration of the charge). Since an upper bound is set on this velocity for technical reasons, for comparatively large bodies with a diameter of several hundred meters or more, whose neutralization requires accordingly higher charge powers, the effect in question yields, apparently, a small increase in energy release in late detection. Protection against collision with such bodies, if they are already near the Earth, is a difficult problem, irrespective of the method of neutralizing action. As far as bodies of the Tunguska-meteorite type are concerned, allowance for the effect yields appreciable results [6]. For example, mini-comets of diameter 30 m can be evaporated completely using a massive 6-ton body without any charge for a velocity of 60 km/sec. In the case of partial destruction of bodies in the Earth's field, the ranges of withdrawal of HSOs with allowance for corrections to the enhancement of the shock wave prove to be nearly k times wider than those without allowance for them.

Also, it is expedient to use the "velocity" factor described under the conditions where it is energy-profitable for fully environmental reasons in realizing large energy release with smaller charges, since the level of radiation hazard occurring in the vicinity of the Earth is reduced.

NOTATION

a, semiaxis of a hyperbola, km; c, area constant, km^2/sec; d, HSO diameter, m; D, velocity of the shock wave, m/sec; e, eccentricity of the orbit; E_0, energy of explosion of a stationary charge, Mton; E_d and E_0', energies of complete and partial destruction of an HSO, Mtons; E_1 and E_k, kinetic energies transferred to the ejected substance and the undestroyed body, Mtons; **F**, explosion-force vector, N; g, g_E, and g_{r_0}, free fall accelerations at an arbitrary point, on the surface of the Earth, and at a distance r_0 from its center, m/sec^2; G, characteristics of destruction; I_0, characteristic mechanical momentum, kg·m/sec; I_f, true value of the total momentum of an HSO, kg·m/sec; I_f', dimensionless total momentum of an HSO; k, kinematic parameter; L, deflection of the HSO trajectory from the Earth's surface, km; M, HSO mass, kg; M_E, Earth's mass, kg; M_0, M_f, and M_1, initial, final, and ejected mass of the body, kg; m, relative mass of an undestroyed body; n, polytropic index of an expanding gas; N, characteristics of motion of the ejected substance and the remaining body; p, orbit parameter, km; P, P_v, and P_{v0}, pressure on the shock-wave front and pressures of vapor efficiency in the comet nucleus at distances of r and 1 au from the Sun, N/m^2; r, geocentric distance, m; $\mathbf{r}_0$, vector determining the initial position of an HSO, km; $r_{\min}$, minimum distance from the HSO to the center of the Earth, km; R_E, Earth's radius, km; R, distance from the epicenter of explosion, m; R_f, distance from the epicenter of explosion to the destruction boundary, m; R_0, radius of the exterior charge surface, m; R_n, radius of the comet nucleus, m; S_n, cross-sectional area of the comet nucleus, m^2; T and T_0, surface temperatures of the comet nucleus at the subsolar point at distances of r and 1 au from the Sun, K; t, time, sec; t_f, instant of completion of the explosion, sec; u and u_x, u_y, average rate of ejection of the HSO substance and its projections onto the X and Y axes, m/sec; u^*, characteristic velocity analogous to the velocity of sound in a condensed substance, m/sec; V and V_x, V_y, velocity of the body and its projections onto the X and Y axes, m/sec; V_0, initial geocentric velocity of an HSO, km/sec; V_0', initial geocentric velocity of an HSO with allowance for explosion momentum, km/sec; ΔV, increment in the HSO velocity, m/sec; v_0 and v, specific volumes of the medium in front of the shock wave and behind it, m^3/kg; w, velocity of the charge in relation to the HSO, km/sec; X and Y, coordinate axes; x, y, corresponding coordinates of an HSO, km; α, parameter characterizing the energy constant; β, destruction parameter; δ, parameter characterizing the initial distance from the HSO to the Earth; γ, gravitation constant of the Earth, N·m^2/kg^2; ε_d, destruction energy, J/kg; ζ, part of the explosion energy transferred to the ejected substance; η, parameter dependent on the polytropic index of an expanding gas; κ, relative velocity of the charge; λ, dimensionless parameter; ξ, coefficient of enhancement of the shock wave; ρ_0, initial density of the medium, kg/m^3; χ, fraction of the explosion energy gone into the shock wave; φ, polar coordinate of an HSO, rad; φ_0, true anomaly of the explosion point, rad; ψ, angle between the vectors $\mathbf{r}_0$ and $\mathbf{F}$, rad; ω, angle between the normal at a point on the shock-wave surface and the direction of motion of the charge (integration constant), rad. Subscripts: 0, initial; f, final; 1, ejected substance; d, destruction; min, minimum; n, nucleus; v, vapor.

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